

VECTOR MODELS FOR DARK ENERGY

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We explore the possibility that the present stage of accelerated expansion of the universe is due to the presence of a cosmic vector field. We show that vector theories allow for the generation of an accelerated phase without the introduction of potential terms or unnatural scales in the Lagrangian. We propose a particular model with the same number of parameters as Λ CDM and excellent fits to SNIa data. The model is scaling during radiation era, with natural initial conditions, thus avoiding the cosmic coincidence problem. Upcoming observations will be able to clearly discriminate it from standard Λ CDM cosmology.

The fact that today the dark energy density is comparable to the matter energy density poses one of the most important problems in order to find viable models of dark energy. Indeed, to achieve this, most of the models, not only the cosmological constant, but also those based on scalar fields such as quintessence or k-essence, and modified gravity theories such as $f(R)$, DGP, etc, require the introduction of unnatural scales either in their Lagrangians or in their initial conditions. This is the so called *cosmic coincidence problem* (see¹ and references therein).

Therefore, we would like to find a model without dimensional scales (apart from Newton's constant G), with the same number of parameters as Λ CDM, with natural initial conditions and with good fits to observations. In addition, the model should be stable under small perturbations. We will show that vector models can do the job².

Let us start by writing the action of our vector-tensor theory of gravity containing only two fields and two derivatives and without potential terms (see³ for previous works on vector models for dark energy with potential terms):

$$S = \int d^4x \sqrt{-g} \left(-\frac{R}{16\pi G} - \frac{1}{2} \nabla_\mu A_\nu \nabla^\mu A^\nu + \frac{1}{2} R_{\mu\nu} A^\mu A^\nu \right) \quad (1)$$

Notice that the theory contains no free parameters, the only dimensional scale being the Newton's constant. The numerical factor in front of the vector kinetic terms can be fixed by the field normalization. Also notice that $R_{\mu\nu} A^\mu A^\nu$ can be written as a combination of derivative terms as $\nabla_\mu A^\mu \nabla_\nu A^\nu - \nabla_\mu A^\nu \nabla_\nu A^\mu$.

The classical equations of motion derived from the action in (1) are the Einstein's and vector field equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G (T_{\mu\nu} + T_{\mu\nu}^A) \quad (2)$$

$$\square A_\mu + R_{\mu\nu} A^\nu = 0 \quad (3)$$

where $T_{\mu\nu}$ is the conserved energy-momentum tensor for matter and radiation and $T_{\mu\nu}^A$ is the energy-momentum tensor coming from the vector field. For the simplest isotropic and homogeneous flat cosmologies, we assume that the spatial components of the vector field vanish, so that $A_\mu = (A_0(t), 0, 0, 0)$ and that the space-time geometry will be given by:

$$ds^2 = dt^2 - a^2(t) \delta_{ij} dx^i dx^j, \quad (4)$$

For this metric (3) reads:

$$\ddot{A}_0 + 3H\dot{A}_0 - 3[2H^2 + \dot{H}]A_0 = 0 \quad (5)$$

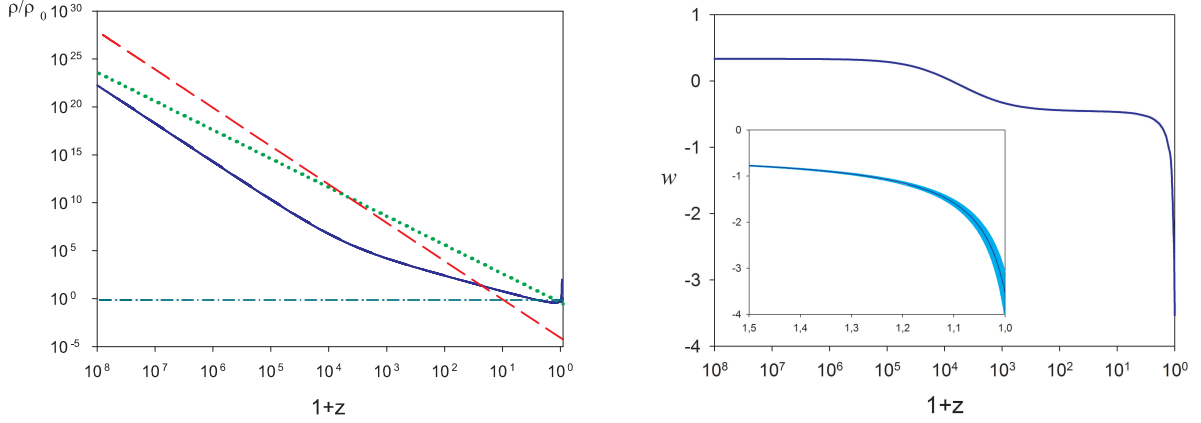


Figure 1: (Left) Evolution of energy densities for the best fit model. Dashed (red) for radiation, dotted (green) for matter and solid (blue) for vector dark energy. We show also for comparison the cosmological constant density in dashed-dotted line. (Right) Evolution of dark energy equation of state for the best fit model. The lower panel shows the 1σ confidence interval.

Assuming that the universe has gone through radiation and matter phases in which the contribution from dark energy was negligible, we can easily solve this equation in those periods. In that case, the above equation has a growing and a decaying solution:

$$A_0(t) = A_0^+ t^{\alpha_+} + A_0^- t^{\alpha_-} \quad (6)$$

with A_0^\pm constants of integration and $\alpha_\pm = -(1 \pm 1)/4$ in the radiation era, and $\alpha_\pm = (-3 \pm \sqrt{33})/6$ in the matter era. On the other hand, the (00) component of Einstein's equations reads:

$$H^2 = \frac{8\pi G}{3} \left[\sum_{\alpha=M,R} \rho_\alpha + \rho_A \right] \quad (7)$$

where the vector energy density is given by:

$$\rho_A = \frac{3}{2} H^2 A_0^2 + 3H A_0 \dot{A}_0 - \frac{1}{2} \dot{A}_0^2 \quad (8)$$

Using the growing mode solution from (6), we obtain $\rho_A = \rho_{A0} a^\kappa$ with $\kappa = -4$ in the radiation era and $\kappa = (\sqrt{33} - 9)/2 \simeq -1.63$ in the matter era. Thus, the energy density of the vector field starts scaling as radiation at early times, so that $\rho_A/\rho_R = \text{const}$. However, when the universe enters its matter era, ρ_A starts growing relative to ρ_M eventually overcoming it at some point, in which the dark energy vector field would become the dominant component (see Fig. 1). Notice that since A_0 is essentially constant during radiation era, solutions do not depend on the precise initial time at which we specify it. Thus, once the present value of the Hubble parameter H_0 and the constant A_0 during radiation (which fixes the total matter density Ω_M) are specified, the model is completely determined, i.e. this model contains the same number of parameters as Λ CDM, which is the minimum number of parameters of a cosmological model with dark energy. As seen from Fig.1 the evolution of the universe ends at a finite time t_{end} with a singularity in which $a \rightarrow a_{\text{end}}$ with a_{end} finite, $A_0(t_{\text{end}}) = M_P/(4\sqrt{\pi})$, $\rho_{DE} \rightarrow \infty$ and $p_{DE} \rightarrow -\infty$.

We can also calculate the effective equation of state for dark energy as:

$$w_{DE} = \frac{p_A}{\rho_A} = \frac{-3 \left(\frac{5}{2} H^2 + \frac{4}{3} \dot{H} \right) A_0^2 + H A_0 \dot{A}_0 - \frac{3}{2} \dot{A}_0^2}{\frac{3}{2} H^2 A_0^2 + 3H A_0 \dot{A}_0 - \frac{1}{2} \dot{A}_0^2} \quad (9)$$

	VCDM Gold	Λ CDM Gold	VCDM SNLS	Λ CDM SNLS
Ω_M	$0.388^{+0.023}_{-0.024}$	$0.309^{+0.039}_{-0.037}$	$0.388^{+0.022}_{-0.020}$	$0.263^{+0.038}_{-0.036}$
w_0	$-3.53^{+0.46}_{-0.57}$	-1	$-3.53^{+0.44}_{-0.48}$	-1
A_0 ($10^{-4} M_P$)	$3.71^{+0.022}_{-0.026}$	—	$3.71^{+0.020}_{-0.024}$	—
z_T	$0.265^{+0.011}_{-0.012}$	$0.648^{+0.101}_{-0.095}$	$0.265^{+0.010}_{-0.012}$	$0.776^{+0.120}_{-0.108}$
t_0 (H_0^{-1})	$0.926^{+0.026}_{-0.023}$	$0.956^{+0.035}_{-0.032}$	$0.926^{+0.022}_{-0.022}$	$1.000^{+0.041}_{-0.037}$
t_{end} (H_0^{-1})	$0.976^{+0.018}_{-0.014}$	—	$0.976^{+0.015}_{-0.013}$	—
χ^2_{min}	172.9	177.1	115.8	111.0

Table 1: Best fit parameters with 1σ intervals for the vector model (VCDM) and the cosmological constant model (Λ CDM) for the Gold (157 SNe) and SNLS (115 SNe) data sets. w_0 denotes the present equation of state of dark energy. A_0 is the constant value of the vector field component during radiation. z_T is the deceleration-aceleration transition redshift. t_0 is the age of the universe in units of the present Hubble time. t_{end} is the duration of the universe in the same units.

Again, using the approximate solutions in (6), we obtain: $w_{DE} = 1/3$ in the radiation era and $w_{DE} \simeq -0.457$ in the matter era. As shown in Fig. 1, the equation of state can cross the so called phantom divide, so that we can have $w_{DE}(z=0) < -1$.

In order to confront the predictions of the model with observations of high-redshift supernovae type Ia, we have carried out a χ^2 statistical analysis for two supernovae datasets, namely, the Gold set⁴, containing 157 points with $z < 1.7$, and the more recent SNLS data set⁵, comprising 115 supernovae but with lower redshifts ($z < 1$). In Table 1 we show the results for the best fit together with its corresponding 1σ intervals for the two data sets. We also show for comparison the results for a standard Λ CDM model. We see that the vector model (VCDM) fits the data considerably better than Λ CDM (in more than 2σ) in the Gold set, whereas the situation is reversed in the SNLS set. This is just a reflection of the well-known 2σ tension⁶ between the two data sets. Compared with Λ CDM, we see that VCDM favors a younger universe (in H_0^{-1} units) with larger matter density. In addition, the deceleration-aceleration transition takes place at a lower redshift in the VCDM case. The present value of the equation of state with $w_0 = -3.53^{+0.46}_{-0.57}$ which clearly excludes the cosmological constant value -1 . Future surveys⁷ are expected to be able to measure w_0 at the few percent level and therefore could discriminate between the two models. We have also compared with other parametrizations for the dark energy equation of state⁸. Since our one-parameter fit has a reduced chi-squared: $\chi^2/d.o.f = 1.108$, VCDM provides the best fit to date for the Gold data set.

We see that unlike the cosmological constant case, throughout radiation era $\rho_{DE}/\rho_R \sim 10^{-6}$ in our case. Moreover the scale of the vector field $A_0 = 3.71 \times 10^{-4} M_P$ in that era is relatively close to the Planck scale and could arise naturally in the early universe without the need of introducing extremely small parameters.

In order to study the model stability we have considered the evolution of metric and vector field perturbations. Thus, we obtain the dispersion relation and the propagation speed of scalar, vector and tensor modes. For all of them we obtain $v = (1 - 16\pi G A_0^2)^{-1/2}$ which is real throughout the universe evolution, since the value $A_0^2 = (16\pi G)^{-1}$ exactly corresponds to that at the final singularity. Therefore the model does not exhibit exponential instabilities. As shown in⁹, the fact that the propagation speed is faster than c does not necessarily implies inconsistencies with causality. We have also considered the evolution of scalar perturbations in the vector field generated by scalar metric perturbations during matter and radiation eras, and found that the energy density contrast $\delta\rho_A/\rho_A$ is constant on super-Hubble scales, whereas it oscillates with growing amplitude as a^2 in the radiation era and as $\sim a^{0.3}$ in the matter era for sub-Hubble scales. Therefore again, we do not find exponentially growing modes.

If we are interested in extending the applicability range of the model down to solar system scales then we should study the corresponding post-Newtonian parameters (PPN). We can see that for the model in (1), the static PPN parameters agree with those of General Relativity¹⁰, i.e. $\gamma = \beta = 1$. For the parameters associated to preferred frame effects we get: $\alpha_1 = 0$ and $\alpha_2 = 8\pi A_\odot^2/M_P^2$ where A_\odot^2 is the norm of the vector field at the solar system scale. Current limits $\alpha_2 \lesssim 10^{-4}$ (or $\alpha_2 \lesssim 10^{-7}$ for static vector fields during solar system formation) then impose a bound $A_\odot^2 \lesssim 10^{-5}(10^{-8}) M_P^2$. In order to determine whether such bounds conflict with the model predictions or not, we should know the predicted value of the field at solar system scales, which in principle does not need to agree with the cosmological value. Indeed, A_\odot^2 will be determined by the mechanism that generated this field in the early universe characterized by its primordial spectrum of perturbations, and the subsequent evolution in the formation of the galaxy and solar system. Another potential difficulty arising generically in vector-tensor models is the presence of negative energy modes for perturbations on sub-Hubble scales. They are known to lead to instabilities at the quantum level, but not necessarily at the classical level as we have shown previously. In any case, the model proposed is not intended as a quantum theory of the gravitational interaction, which would be beyond the scope of this work.

In conclusion, vector theories offer an accurate phenomenological description of dark energy in which fine tuning problems could be easily avoided.

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References

1. E.J. Copeland, M. Sami and S. Tsujikawa, *Int. J. Mod. Phys. D*, **15**, 1753 (2006)
2. J. Beltrán-Jiménez and A.L. Maroto, arXiv:0801.1486 [astro-ph]
3. V.V. Kiselev, *Class. Quant. Grav.* **21**: 3323, (2004); C. Armendariz-Picon, *JCAP* **0407**: 007, (2004); C.G. Boehmer and T. Harko, *Eur. Phys. J.* **C50**: 423, (2007); M. Novello, et al. *Phys. Rev.* **D69**: 127301, (2004); T. Koivisto and D.F. Mota, e-Print: arXiv:0707.0279
4. A.G. Riess et al. *Astrophys.J.* **607**, 665 (2004)
5. P. Astier et al., *Astron. Astrophys.* **447**: 31-48, (2006).
6. S. Nesseris and L. Perivolaropoulos, *JCAP* **0702**: 025, (2007).
7. R. Trotta and R. Bower, *Astron. Geophys.* **47**: 4:20-4:27, (2006)
8. R. Lazkoz, S. Nesseris, L. Perivolaropoulos, *JCAP* **0511**:010, (2005)
9. E. Babichev, V. Mukhanov and A. Vikman, *JHEP* **0802**, 101 (2008)
10. C. Will, *Theory and experiment in gravitational physics*, Cambridge University Press, (1993)